Section 3.1 Meaning and Properties of Fractions

1. Definition of a Fraction: A fraction is any number that can be put in the form $\frac{a}{b}$ where a and b are integers and b is not 0. The numerator of the fraction is "a" and the denominator is "b". A proper fraction is a fraction in which the numerator is less than the denominator. An improper fraction is a fraction in which the numerator is greater than or equal to the denominator.

Example: Answer each of the following.

a. Give an example of a proper fraction.
b. Give an example of an improper fraction.
c. What are the integers?
d. Is $\frac{7}{7}$ a proper fraction or an improper fraction?

2. Equivalent Fractions: Fractions that represent the same number are said to be equivalent.

Example: For each fraction below, name an equivalent fraction.

a. $\frac{4}{8} = \frac{8}{16} = \frac{12}{24} = \frac{1}{2}$

b. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{24}$

c. $\frac{10}{2} = \frac{20}{4} = \frac{30}{6} = \frac{5}{1} = 5$

3. Property One for Fractions: If a, b and c are integers and b and c are not 0, then it is true that

$$\frac{a}{b} = \frac{a \bullet c}{b \bullet c}$$

This property is often used to rewrite a given fraction as an equivalent fraction with a specific denominator. This is particularly useful when rewriting a several fractions with a common denominator.

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Example: Rewrite each fraction as an equivalent fraction with the given denominator.

a. $\frac{3}{4} = \frac{}{20}$ $\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20}$

b. $\frac{5}{7}$, denominator of 35 $\frac{5}{7} = \frac{}{35}$, $\frac{5}{7} = \frac{5 \cdot 5}{7 \cdot 5} = \frac{25}{35}$

c. $\frac{9}{11} = \frac{}{33}$ $\frac{9}{11} = \frac{9 \cdot 3}{11 \cdot 3} = \frac{27}{33}$

d. $\frac{8}{10}$, denominator of 25 $\frac{8}{10} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 5} = \frac{2 \cdot 2}{5} = \frac{2 \cdot 2 \cdot 5}{5 \cdot 5} = \frac{20}{25}$

$\frac{8}{10} = \frac{}{25}$

4. Property Two for Fractions: If a, b and c are integers and b and c are not 0, then it is true that

$$\frac{a}{b} = \frac{a \div c}{b \div c}$$

This property is often used to reduce a given fraction to lowest terms. To reduce to lowest terms, the number "c" is the greatest common factor for the numerator and denominator.

Example: Reduce each fraction to lowest terms.

a. $\frac{30}{45} = \frac{30 \div 15}{45 \div 15} = \frac{2}{3}$

b. $\frac{10}{14} = \frac{10 \div 2}{14 \div 2} = \frac{5}{7}$

c. $\frac{22}{33} = \frac{22 \div 11}{33 \div 11} = \frac{2}{3}$

d. $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$

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5. The Number 1 and Fractions: If a is any number, then it is true that

$$\frac{a}{1} = a \quad \text{and} \quad \frac{a}{a} = 1.$$

Example: Simplify each expression.

a. $\frac{7}{1} = 7$

b. $\frac{16}{16} = 1$

c. $\frac{-3}{-3} = 1$

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Section 3.2 Prime Numbers, Factors, and Reducing to Lowest Terms

1. Prime Numbers: A prime number is any whole number greater than 1 that has exactly two divisors: itself and 1. (A number is a divisor of another number if it divides that number without a remainder.)

Example: List the prime numbers that are smaller than 20.

2, 3, 5, 7, 11, 13, 17, 19

Primes under 100 → 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 83, 89, 97

2. Divisibility Tests: The following divisibility tests will help you in deciding what numbers are factors of the given number.

- Divisibility test for 2: A given number is divisible by 2 if it ends in an even digit.
- Divisibility test for 3: A given number is divisible by 3 if the sum of the digits in the number is divisible by 3.
- Divisibility test for 5: A given number is divisible by 5 if the last digit is 5 or 0.

Example: For each given number, apply the tests given above to determine if the number is divisible by 2, 3 or 5.

a. 105 – Ends in 5, therefore divisible by 5
– 1+0+5=6, 6 is divisible by 3, therefore 105 is divisible by 3
– not even, therefore 105 is NOT divisible by 2

b. 362 – Even number, therefore 362 is divisible by 2
– 3+6+2=11, 11 is Not divisible by 3, therefore 362 is Not divisible by 3
– Ends in 2, therefore is Not divisible by 5

c. 150 – Ends in 0, therefore is divisible by ~~by~~ 5
– Even number, therefore 150 is divisible by 2
– 1+5+0=6, 6 is divisible by 3, therefore 150 is divisible by 3

d. 2,304 – Even number, therefore is divisible by 2
– 2+3+0+4=9, 9 is divisible by 3, therefore 2,304 is divisible by 3
– Ends in 4, therefore 2,304 is Not divisible by 5

e. 34,005 – ENDS in 5, therefore 34,005 is divisible by 5
– not even, therefore 34,005 is Not Divisible by 2
– 3+4+0+0+5=12, 12 is divisible by 3, therefore 34,005 is divisible by 3.

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3. Writing the Prime Factorization of a Number: To write the prime factorization of a number, write the number as a product of prime numbers.

Example: Which of the following represents a prime factorization?

a. $12 = 6 \bullet 2$, NO , 6 is not prime

b. $12 = 2^2 \bullet 3$, Yes

c. $15 = 5 \bullet 3$, Yes

d. $22 = 11 \bullet 2$, Yes

e. $20 = 4 \bullet 5$, No , 4 is not prime

You can use a factor tree to find a prime factorization. Express the given number as the product of two numbers. Then express each of those as the product of two numbers. Continue this process until the numbers no longer factor--you will then have the prime factors.

Example: Find the prime factorization for each of the following numbers:

a. 18 $= 2 \cdot 3 \cdot 3$
$= 2 \cdot 3^2$

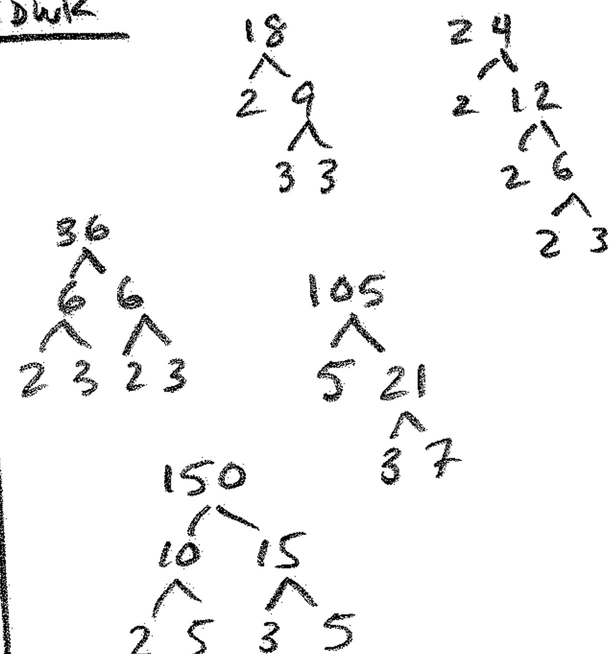
b. 24 $= 2 \cdot 2 \cdot 2 \cdot 3$
$= 2^3 \cdot 3$

c. 36 $= 2 \cdot 2 \cdot 3 \cdot 3$
$= 2^2 \cdot 3^2$

d. 105 $= 3 \cdot 5 \cdot 7$

e. 150 $= 2 \cdot 3 \cdot 5 \cdot 5$
$= 2 \cdot 3 \cdot 5^2$

15 DWK



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4. Reducing a fraction to lowest terms: A fraction is said to be in lowest terms if the numerator and denominator have no common factors other than 1.

To reduce a fraction to lowest terms, divide the numerator and the denominator by all of the factors that they have in common. If you do not know the common factors, write the prime factorization of the numerator and denominator, then divide out the common factors.

Example: Reduce the given fractions to lowest terms.

a. $\frac{21}{35} = \frac{3 \cdot \cancel{7}}{5 \cdot \cancel{7}} = \frac{3}{5}$

b. $\frac{18}{24} = \frac{\cancel{2} \cdot 3 \cdot \cancel{3}}{\cancel{2} \cdot 2 \cdot 2 \cdot \cancel{3}} = \frac{3}{2 \cdot 2} = \frac{3}{4}$

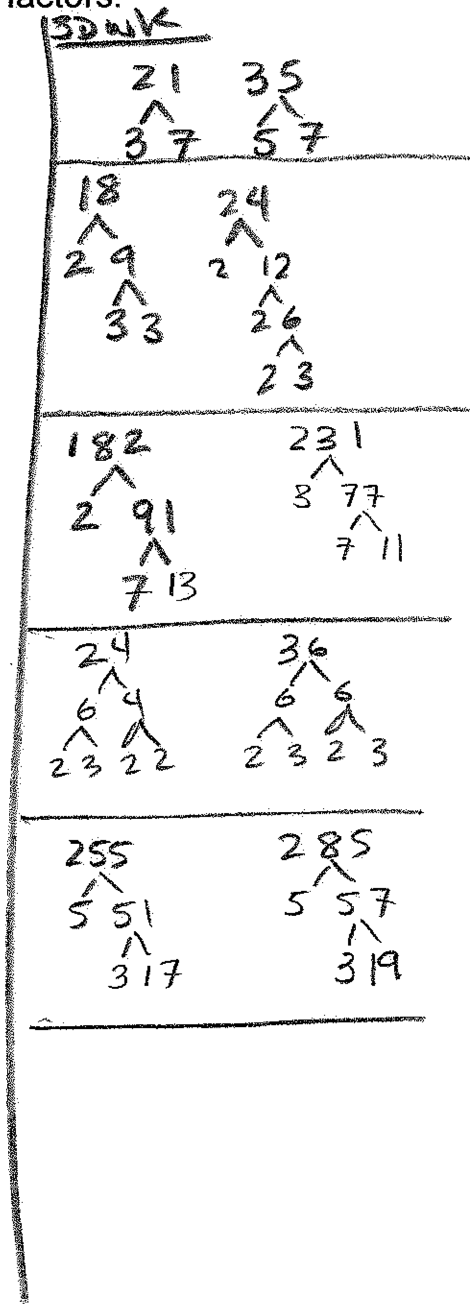
c. $\frac{105}{150} = \frac{3 \bullet 5 \bullet 7}{2 \bullet 3 \bullet 5 \bullet 5} = \frac{3 \bullet 5}{3 \bullet 5} \bullet \frac{7}{2 \bullet 5} = \frac{7}{10}$

d. $\frac{182}{231} = \frac{2 \cdot 7 \cdot 13}{3 \cdot 7 \cdot 11} = \frac{2 \cdot 13}{3 \cdot 11} = \frac{26}{33}$

e. $\frac{24x}{36x} = \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3} \cdot \cancel{x}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 3 \cdot \cancel{x}} = \frac{2}{3}$

f. $\frac{255a^2b^2c}{285ab^2c^3} = \frac{\cancel{3} \cdot \cancel{5} \cdot 17 \cdot \cancel{a} \cdot a \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{c}}{\cancel{3} \cdot \cancel{5} \cdot 19 \cdot \cancel{a} \cdot \cancel{b} \cdot \cancel{b} \cdot \cancel{c} \cdot c \cdot c}$

$= \frac{17 \cdot a}{19 \cdot c \cdot c} = \frac{17a}{19c^2}$



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Example: A family spends \$2200 monthly for a mortgage payment and has a monthly income of \$5500. What fraction of the total income is used for the mortgage payment? Express your answer in lowest terms.

Fraction of total income used for the mortgage payment $= \dfrac{\$2,200}{\$5,500}$

$= \dfrac{2 \cdot \cancel{11} \cdot \cancel{100}}{5 \cdot \cancel{11} \cdot \cancel{100}}$

$= \dfrac{2}{5}$

ANS: $\frac{2}{5}$ of the total income is spent on the mortgage payment.

SDNK
2,200 → 22, 100; 22 → 2, 11
5,500 → 55, 100; 55 → 5, 11

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3.3 Multiplication with Fractions and the Area of a Triangle

1. Rule for Multiplying Fractions: The product of two fractions is the fraction whose numerator is the product of the two numerators and whose denominator is the product of the two denominators. In symbols, if a,b,c and d represent any numbers and b and d are not zero, then

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$$

Example: Multiply the given fractions.

a. $\frac{2}{7} \bullet \frac{3}{5} = \frac{2 \cdot 3}{7 \cdot 5} = \frac{6}{35}$

b. $\frac{4}{5} \bullet \frac{6}{11} = \frac{4 \cdot 6}{5 \cdot 11} = \frac{24}{55}$

c. $-\frac{3}{10} \bullet \frac{7}{16} = -\frac{3 \cdot 7}{10 \cdot 16} = -\frac{21}{160}$

d. $-\frac{4}{19}\left(-\frac{5}{9}\right) = +\frac{4 \cdot 5}{19 \cdot 9} = \frac{20}{171}$

$\underline{\text{SDWK}}$

$$\begin{array}{r} \scriptstyle 8 \\ 19 \\ \times 9 \\ \hline 171 \end{array}$$

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2. Using the Commutative and Associative Properties of Multiplication with Fractions

When multiplying fractions, the commutative and associative properties of multiplication are often useful in simplifying.

Example: Multiply the given fractions.

a. $3\left(\frac{1}{3}\bullet\frac{4}{5}\right)=\left(3\bullet\frac{1}{3}\right)\bullet\frac{4}{5}=\left(\frac{3}{1}\bullet\frac{1}{3}\right)\bullet\frac{4}{5}=\frac{4}{5}$

b. $\frac{2}{7}\left(\frac{3}{5}x\right) = \left(\frac{2}{7}\cdot\frac{3}{5}\right)\cdot x = \frac{2\cdot 3}{7\cdot 5}\cdot x = \frac{6}{35}x$ or $\frac{6x}{35}$

c. $5\left(\frac{3}{4}x\right) = \left(\frac{5}{1}\cdot\frac{3}{4}\right)\cdot x = \frac{5\cdot 3}{1\cdot 4}\cdot x = \frac{15}{4}x$ or $\frac{15x}{4}$

d. $-\frac{3}{10}\left(\frac{7}{16}x\right) = \left(-\frac{3}{10}\cdot\frac{7}{16}\right)\cdot x = \left(-\frac{3\cdot 7}{10\cdot 16}\right)\cdot x = \frac{-21}{160}x$ or $\frac{-21x}{160}$

e. $-\frac{4}{19}\left(-\frac{5}{9}x\right) = \left(+\frac{4}{19}\cdot\frac{5}{9}\right)\cdot x = \frac{4\cdot 5}{19\cdot 9}\cdot x = \frac{20}{171}x$ or $\frac{20x}{171}$

3. Dividing out Common Factors Before Multiplying: When multiplying fractions factor the numerator and denominator and then divide out common factors **before** multiplying. Your answer will then always be in lowest form. This step, where you divide out common factors before you multiply, is worth points on every multiplication of fractions problem, and if you omit the step you will lose those points.

Example: Multiply the given fractions. Divide out any common factors before you multiply.

a. $\frac{4}{9}\bullet\frac{21}{25} = \frac{2\cdot 2\cdot\cancel{3}\cdot 7}{3\cdot\cancel{3}\cdot 5\cdot 5} = \frac{2\cdot 2\cdot 7}{3\cdot 5\cdot 5} = \frac{28}{75}$

SD WK

21 → 3 7   25 → 5 5

$2\cdot 2\cdot 7$
$= 4\cdot 7$
$= 28$

$3\cdot 5\cdot 5$
$= 15\cdot 5$
$= 75$

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SDWK

14: 2 · 7; 15: 3 · 5; 20: 2 · 10, 10: 2 · 5; 49: 7 · 7

22: 2 · 11; 35: 5 · 7; 28: 4 · 7, 4: 2 · 2; 55: 5 · 11

14: 2 · 7; 39: 3 · 13; 42: 6 · 7, 6: 2 · 3; 63: 7 · 9, 9: 3 · 3

22: 2 · 11; 8: 2 · 4, 4: 2 · 2; 9: 3 · 3; 35: 5 · 7

b. $\frac{14}{15}\bullet\frac{20}{49} = \frac{2\cdot\cancel{7}\cdot 2\cdot 2\cdot\cancel{5}}{3\cdot\cancel{5}\cdot\cancel{7}\cdot 7} = \frac{2\cdot 2\cdot 2}{3\cdot 7} = \frac{8}{21}$

c. $\frac{22}{35}\bullet\frac{28}{55} = \frac{2\cdot\cancel{11}\cdot 2\cdot 2\cdot\cancel{7}}{5\cdot\cancel{7}\cdot 5\cdot\cancel{11}} = \frac{2\cdot 2\cdot 2}{5\cdot 5} = \frac{8}{25}$

d. $\frac{14}{39}\bullet\frac{42}{63} = \frac{2\cdot 7\cdot 2\cdot\cancel{3}\cdot\cancel{7}}{3\cdot 13\cdot 3\cdot\cancel{3}\cdot\cancel{7}} = \frac{2\cdot 2\cdot 7}{3\cdot 3\cdot 13} = \frac{28}{117}$

20: 5 · 4, 4: 2 · 2; 14: 2 · 7; 9: 3 · 3

e. $-\frac{3}{8}\bullet\frac{7}{9}\bullet\frac{22}{35} = -\frac{\cancel{3}\cdot\cancel{7}\cdot\cancel{2}\cdot 11}{2\cdot 2\cdot\cancel{2}\cdot\cancel{3}\cdot 3\cdot 5\cdot\cancel{7}} = -\frac{11}{2\cdot 2\cdot 3\cdot 5} = \frac{-11}{60}$

16: 4 · 4, 4: 2 · 2, 4: 2 · 2; 25: 5 · 5; 33: 3 · 11

f. $\frac{2}{3}\bullet\frac{7}{20}\bullet\frac{9}{14} = \frac{\cancel{2}\cdot\cancel{7}\cdot\cancel{3}\cdot 3}{\cancel{3}\cdot 2\cdot 2\cdot 5\cdot\cancel{2}\cdot\cancel{7}} = \frac{3}{2\cdot 2\cdot 5} = \frac{3}{20}$

22: 2 · 11

g. $-\frac{4}{5}\left(-\frac{11}{16}\right)\left(-\frac{25}{33}\right) = -\frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{11}\cdot\cancel{5}\cdot 5}{\cancel{5}\cdot\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2\cdot 3\cdot\cancel{11}} = -\frac{5}{2\cdot 2\cdot 3} = -\frac{5}{12}$

33: 3 · 11; 15: 3 · 5; 9: 3 · 3

h. $\frac{5}{9}\bullet\frac{22}{33}\bullet\frac{11}{15} = \frac{\cancel{5}\cdot 2\cdot\cancel{11}\cdot 11}{3\cdot 3\cdot 3\cdot\cancel{11}\cdot 3\cdot\cancel{5}} = \frac{2\cdot 11}{3\cdot 3\cdot 3\cdot 3} = \frac{22}{81}$

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SDWK

$4 \to 2\ 2$

i. $\frac{x}{y} \bullet \frac{y^2}{x} = \frac{\cancel{x} \cdot \cancel{y} \cdot y}{\cancel{y} \cdot \cancel{x} \cdot 1} = \frac{y}{1} = y$

j. $\frac{2xy}{z^2} \bullet \frac{z}{4xy^2} = \frac{\cancel{2} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{z} \cdot 1}{z \cdot \cancel{z} \cdot \cancel{2} \cdot 2 \cancel{x} \cdot \cancel{y} \cdot y} = \frac{1}{z \cdot 2 \cdot y} = \frac{1}{2yz}$

SDWK

$2 \cdot 2 \cdot 2 = 2 \cdot 4 = 8$

$5 \cdot 5 \cdot 5 = 25 \cdot 5 = 125$

$4 \to 2\ 2$

$8 \to 2\ 4 \to 2\ 2$

4. Multiplying with Fractions and Exponents: When simplifying expressions that contain fractions and exponents, use the order of operations agreement and the exponent rules.

Example: Simplify the following expressions.

a. $\left(\frac{1}{4}\right)^2 = \frac{1}{4} \cdot \frac{1}{4} = \frac{1 \cdot 1}{4 \cdot 4} = \frac{1}{16}$

b. $\left(-\frac{2}{5}\right)^3 = \left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right) = -\frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5} = -\frac{8}{125}$

c. $\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)^2 = \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{1 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 3 \cdot 3} = \frac{1}{3 \cdot 3} = \frac{1}{9}$

d. $-\left(\frac{1}{4}\right)^2\left(\frac{8}{11}\right) = -\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{8}{11} = -\frac{1 \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}}{2 \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 11} = -\frac{1}{2 \cdot 11}$

$= -\frac{1}{22}$

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5. Interpreting "of" When Used with Fractions: The word "of" when used in connection with fractions indicates multiplication.

Example: Simplify

a. Find $\frac{1}{2}$ of $\frac{2}{5}$.

$\frac{1}{2} \cdot \frac{2}{5} = \frac{1 \cdot \cancel{2}}{\cancel{2} \cdot 5} = \frac{1}{5}$

b. Find $\frac{2}{3}$ of $-\frac{3}{8}$.

$\frac{2}{3} \cdot \left(-\frac{3}{8}\right) = -\frac{\cancel{2} \cdot \cancel{3} \cdot 1}{\cancel{3} \cdot \cancel{2} \cdot 2 \cdot 2} = -\frac{1}{2 \cdot 2} = -\frac{1}{4}$

c. What is $\frac{3}{7}$ of -21.

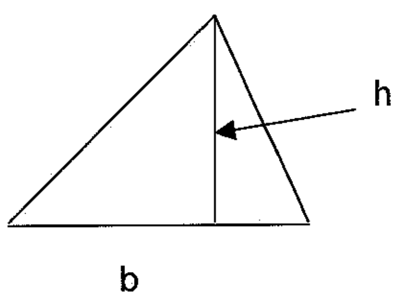
$\frac{3}{7} \cdot \left(-\frac{21}{1}\right) = -\frac{3 \cdot 3 \cdot \cancel{7}}{\cancel{7} \cdot 1} = -\frac{3 \cdot 3}{1} = -\frac{9}{1} = -9$

SDWK

8 → 2 4 → 2 2

21 → 3 7

6. Area of a Triangle:



The area of a triangle is given by the formula

$$A = \frac{1}{2}bh$$

Example: Find the area of a triangle that has a base of 13 inches and a height of 12 inches.

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \cdot \left(\frac{13 \text{ in}}{1}\right)\left(\frac{12 \text{ in}}{1}\right)$

$A = \frac{1 \cdot 13 \cdot 12}{2} \text{ in}^2$

$A = \frac{13 \cdot \cancel{2} \cdot 2 \cdot 3}{1 \cdot \cancel{2}} \text{ in}^2$

$A = \frac{13 \cdot 2 \cdot 3}{1} \text{ in}^2$

$A = 78 \text{ in}^2$

SDWK

12 → 4 3 → 2 2

$13 \times 6 = 78$

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SDWK

3.4 Division with Fractions

1. Rule for Dividing Fractions: If a,b,c and d are integers and neither b,c nor d is zero, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$$

To divide two fractions, multiply the first fraction by the reciprocal of the second.

Example: Divide the given fractions.

a. $\frac{5}{9} \div \frac{10}{1} = \frac{5}{9} \cdot \frac{1}{10} = \frac{\cancel{5} \cdot 1}{3 \cdot 3 \cdot 2 \cdot \cancel{5}} = \frac{1}{3 \cdot 3 \cdot 2} = \frac{1}{18}$

b. $\left(\frac{-27}{1}\right) \div \left(\frac{3}{2}\right) = -\frac{27}{1} \cdot \frac{2}{3} = -\frac{3 \cdot 3 \cdot \cancel{3} \cdot 2}{1 \cdot \cancel{3}} = -\frac{3 \cdot 3 \cdot 2}{1} = -18$

c. $-\frac{x}{y^2} \div \frac{x}{y} = -\frac{x}{y^2} \cdot \frac{y}{x} = -\frac{\cancel{x} \cdot \cancel{y} \cdot 1}{y \cdot \cancel{y} \cdot \cancel{x}} = -\frac{1}{y}$

d. $\left(\frac{-15}{1}\right) \div \left(-\frac{4}{3}\right) = \frac{15}{1} \cdot \frac{3}{4} = \frac{3 \cdot 5 \cdot 3}{1 \cdot 2 \cdot 2} = \frac{45}{4}$

e. $\frac{18}{25} \div \left(-\frac{6}{35}\right) = -\frac{18}{25} \cdot \frac{35}{6} = -\frac{\cancel{2} \cdot 3 \cdot \cancel{3} \cdot \cancel{5} \cdot 7}{5 \cdot \cancel{5} \cdot \cancel{2} \cdot \cancel{3}} = -\frac{3 \cdot 7}{5} = \frac{-21}{5}$

2. Simplifying Fractional Expressions That Contain Multiplication, Division and Exponents: Use the order of operations agreement and the rules for adding, subtracting, multiplying and dividing fractions to simplify.

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Example: Simplify each of the following.

a. $\frac{5}{32} \div \frac{15}{16} \bullet \frac{2}{3} = \frac{5}{32} \cdot \frac{16}{15} \cdot \frac{2}{3} = \frac{\cancel{5} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 1}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{5} \cdot 3} = \frac{1}{3 \cdot 3} = \frac{1}{9}$

b. $\frac{10a^2}{3b} \div \frac{5a}{6b} \bullet \frac{1}{2} = \frac{10a^2}{3b} \cdot \frac{6b}{5a} \cdot \frac{1}{2} = \frac{2 \cdot \cancel{5} \cdot \cancel{a} a \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{b} \cdot 1}{\cancel{3} \cdot \cancel{b} \cdot \cancel{5} \cdot \cancel{a} \cdot \cancel{2} \cdot 1} = \frac{2 \cdot a}{1} = 2a$

c. $-\frac{28}{125} \div \left(\frac{5x}{2}\right) \bullet \frac{1}{x} = -\frac{28}{125} \cdot \frac{2}{5x} \cdot \frac{1}{x} = -\frac{2 \cdot 2 \cdot 7 \cdot 2 \cdot 1}{5 \cdot 5 \cdot 5 \cdot 5 \cdot x \cdot x} = -\frac{56}{625x^2}$

d. $\frac{4}{5} \bullet \frac{5}{3} \div \left(-\frac{8}{9}\right) = -\frac{4}{5} \cdot \frac{5}{3} \cdot \frac{9}{8} = -\frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{5} \cdot \cancel{3} \cdot 3}{\cancel{5} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{2} \cdot 2} = -\frac{3}{2}$

e. $24 \div \left(\frac{2}{5}\right)^2 + 25 \div \left(\frac{5}{6}\right)^2$

$= \frac{24}{1} \div \frac{4}{25} + \frac{25}{1} \div \frac{25}{36}$

$= \frac{24}{1} \cdot \frac{25}{4} + \frac{25}{1} \cdot \frac{36}{25}$

$= \frac{2 \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot 5 \cdot 5}{1 \cdot \cancel{2} \cdot \cancel{2}} + \frac{\cancel{5} \cdot \cancel{5} \cdot 2 \cdot 2 \cdot 3 \cdot 3}{1 \cdot \cancel{5} \cdot \cancel{5}}$

$= \frac{2 \cdot 3 \cdot 5 \cdot 5}{1} + \frac{2 \cdot 2 \cdot 3 \cdot 3}{1}$

$= \frac{150}{1} + \frac{36}{1}$

$= 150 + 36$

$= 186$

f. $9 \div \left(\frac{3}{5}\right)^2 + 25 \div \left(\frac{5}{7}\right)^2$

$= \frac{9}{1} \div \frac{9}{25} + \frac{25}{1} \div \frac{25}{49}$

$= \frac{9}{1} \cdot \frac{25}{9} + \frac{25}{1} \cdot \frac{49}{25}$

$= \frac{\cancel{3} \cdot \cancel{3} \cdot 5 \cdot 5}{1 \cdot \cancel{3} \cdot \cancel{3}} + \frac{\cancel{5} \cdot \cancel{5} \cdot 7 \cdot 7}{1 \cdot \cancel{5} \cdot \cancel{5}}$

$= \frac{25}{1} + \frac{49}{1}$

$= 25 + 49$

$= 74$

SDWK

15 → 5 3; 32 → 2 16 → 2 8 → 2 4 → 2 2; 16 → 2 8 → 2 4 → 2 2

10 → 2 5; 6 → 2 3

28 → 7 4 → 2 2; 125 → 5 25 → 5 5

$28 \times 2 = 56$

25×25: $125 + 500 = 625$

24 → 3 8 → 2 4 → 2 2; 25 → 5 5; 36 → 2 18 → 2 9 → 3 3

$25 \times 6 = 150$

9 → 3 3; 25 → 5 5; 49 → 7 7

35 → 7 5

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

3.5 Addition and Subtraction with Fractions

1. Addition and Subtraction When the Denominators Are the Same: To add or subtract fractions that have the same denominator, add or subtract the numerators to get the numerator of the answer and carry along the common denominator to get the denominator of the answer.

Example: Simplify the following.

a. $\frac{3}{8}+\frac{5}{8}=\frac{8}{8}=1$

b. $\frac{4}{11}+\frac{5}{11}=\frac{4+5}{11}=\frac{9}{11}$

c. $\frac{5}{13}+\frac{8}{13}=\frac{5+8}{13}=\frac{13}{13}=1$

2. Addition and Subtraction When the Denominators Are Not the Same: To add or subtract fractions that do not have the same denominator:

- Find the least common denominator (LCD).
- Rewrite the fractions as equivalent fractions that have the LCD as their denominator.
- Add or subtract the numerators to get the numerator of the answer and carry along the common denominator to get the denominator of the answer.

NOTE: On tests, when adding or subtracting fractions, you must rewrite the fractions as equivalent fractions that have the least common denominator. This step is worth points, and to get the points your denominator must be the **least** common denominator.

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Example: Simplify the following.

a. $\frac{3}{4}+\frac{1}{2}=\frac{?}{4}+\frac{?}{4}$ the LCD is 4

$=\frac{3}{4}+\frac{2}{4}$ rewrite using the LCD and adjusting the numerators

$=\frac{3+2}{4}$ ← missing step!

$=\frac{5}{4}$ add the numerators and carry along the LCD

SDWK

$LCD = 2\cdot 7\cdot 3 = 42$

$14 = 2\cdot 7$

$21 = 3\cdot 7$

14: 2, 7   21: 3, 7

b. $\frac{1}{14}+\frac{2}{21} = \frac{1}{14}\cdot\frac{3}{3}+\frac{2}{21}\cdot\frac{2}{2}$

$=\frac{3}{42}+\frac{4}{42}$

$=\frac{3+4}{42}$

$=\frac{7}{42}$

$=\frac{1\cdot\cancel{7}}{2\cdot 3\cdot\cancel{7}}$

$=\frac{1}{2\cdot 3}$

$=\frac{1}{6}$

SDWK

$LCD = 2\cdot 5\cdot 3$

$LCD = 30$

$10 = 2\cdot 5$

$15 = 3\cdot 5$

25: 5, 5   15: 3, 5   10: 2, 5

c. $\frac{3}{10}+\frac{8}{15}$

$=\frac{3}{10}\cdot\frac{3}{3}+\frac{8}{15}\cdot\frac{2}{2}$

$=\frac{9}{30}+\frac{16}{30}$

$=\frac{9+16}{30}$

$=\frac{25}{30}$

$=\frac{5\cdot\cancel{5}}{2\cdot 3\cdot\cancel{5}}$

$=\frac{5}{2\cdot 3}$

$=\frac{5}{6}$

SDWK

$LCD = 2\cdot 2\cdot 3\cdot 5 = 60$

$12 = 2\cdot 2\cdot 3$

$30 = 2\cdot 3\cdot 5$

12: 2, 6; 6: 2, 3   30: 6, 5; 6: 2, 3   39: 3, 13

d. $\frac{5}{12}+\frac{7}{30}$

$=\frac{5}{12}\cdot\frac{5}{5}+\frac{7}{30}\cdot\frac{2}{2}$

$=\frac{25}{60}+\frac{14}{60}$

$=\frac{25+14}{60}$

$=\frac{39}{60}$

$=\frac{\cancel{3}\cdot 13}{2\cdot 2\cdot\cancel{3}\cdot 5}$

$=\frac{13}{2\cdot 2\cdot 5}$

$=\frac{13}{20}$

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3. Addition and Subtraction If the Fractions Have Different Denominators and You Don't Know the Least Common Denominator (LCD): To "build" the least common denominator, prime factor each denominator, and give each denominator (and numerator) the factors that it is missing from the other denominator.

Example: Simplify the following.

a. $\frac{5}{36}+\frac{7}{50}=\frac{5}{2\bullet 2\bullet 3\bullet 3}+\frac{7}{2\bullet 5\bullet 5}$

$$=\frac{5}{2\bullet 2\bullet 3\bullet 3}\bullet\frac{5\bullet 5}{5\bullet 5}+\frac{7}{2\bullet 5\bullet 5}\bullet\frac{2\bullet 3\bullet 3}{2\bullet 3\bullet 3}$$

$$=\frac{125}{900}+\frac{126}{900}$$

$$=\frac{125+126}{900}$$

missing step

$$=\frac{251}{900}$$

b. $\frac{4}{15}+\frac{2}{27}=\frac{4}{3\cdot 5}+\frac{2}{3\cdot 3\cdot 3}$

$$=\frac{4}{3\cdot 5}\cdot\frac{3\cdot 3}{3\cdot 3}+\frac{2}{3\cdot 3\cdot 3}\cdot\frac{5}{5}$$

$$=\frac{36}{135}+\frac{10}{135}$$

$$=\frac{36+10}{135}$$

$$=\frac{46}{135}$$

SDWK

$36 \to 6, 6 \to 2, 3, 2, 3$

$50 \to 5, 10 \to 2, 5$

$36 = 2\cdot 2\cdot 3\cdot 3$

$50 = 2\cdot 5\cdot 5$

$LCD = 2\cdot 2\cdot 3\cdot 3\cdot 5\cdot 5$

$LCD = 4\cdot 9\cdot 25$

$LCD = 9\cdot 100$

$LCD = 900$

$5\cdot 5\cdot 5 = 25\cdot 5$

$= 125$

$7\cdot 2\cdot 3\cdot 3 = 14\cdot 9$

$= 126$

$15 \to 3, 5$

$27 \to 3, 9 \to 3, 3$

$15 = 3\cdot 5$

$27 = 3\cdot 3\cdot 3$

$LCD = 3\cdot 3\cdot 3\cdot 5$

$LCD = 9\cdot 15$

$LCD = 135$

$46 = 2\cdot 23$

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c. $\frac{20}{51}-\frac{7}{36}=\frac{20}{3\cdot 17}-\frac{7}{2\cdot 2\cdot 3\cdot 3}$

$=\frac{20}{3\cdot 17}\cdot\frac{2\cdot 2\cdot 3}{2\cdot 2\cdot 3}-\frac{7}{2\cdot 2\cdot 3\cdot 3}\cdot\frac{17}{17}$

$=\frac{240}{612}-\frac{119}{612}$

$=\frac{240-119}{612}$

$=\frac{121}{612}$

d. $\frac{7}{54}+\frac{11}{90}=\frac{7}{2\cdot 3\cdot 3\cdot 3}+\frac{11}{2\cdot 3\cdot 3\cdot 5}$

$=\frac{7}{2\cdot 3\cdot 3\cdot 3}\cdot\frac{5}{5}+\frac{11}{2\cdot 3\cdot 3\cdot 5}\cdot\frac{3}{3}$

$=\frac{35}{270}+\frac{33}{270}$

$=\frac{35+33}{270}$

$=\frac{68}{270}$

$=\frac{2\cdot 2\cdot 17}{2\cdot 3\cdot 3\cdot 3\cdot 5}$

$=\frac{2\cdot 17}{3\cdot 3\cdot 3\cdot 5}$

$=\frac{34}{135}$

e. $\frac{8}{35}-\frac{6}{49}=\frac{8}{5\cdot 7}-\frac{6}{7\cdot 7}$

$=\frac{8}{5\cdot 7}\cdot\frac{7}{7}-\frac{6}{7\cdot 7}\cdot\frac{5}{5}$

$=\frac{56}{245}-\frac{30}{245}$

$=\frac{56-30}{245}$

$=\frac{26}{245}$

SDWK

51 = 3 · 17
36 = 2·2·3·3

LCD = 2·2·3·3·17
LCD = 4·9·17
LCD = 36·17
LCD = 612

36×17: 252 + 360 = 612

121 = 11 · 11

54 = 2 · 27 = 2·3·9 = 2·3·3·3
90 = 9 · 10 = 3·3·2·5

54 = 2·3·3·3
90 = 2·3·3·5
LCD = 2·3·3·3·5
LCD = 6·9·5
LCD = 30·9
LCD = 270

68 = 2 · 34 = 2·2·17

3·3·3·5
= 9·15
= 135

35 = 5 · 7
49 = 7 · 7

35 = 5·7
49 = 7·7
LCD = 5·7·7
LCD = 35·7
LCD = 245

$35 \times 7 = 245$

26 = 2 · 13

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3.6 Mixed-Number Notation

1. What is a mixed number? A mixed number is the sum of a whole number and a proper fraction. We leave out the plus sign when we write the number.

Example:

$3+\frac{1}{8}=3\frac{1}{8}$ (In mixed - number notation we leave out the plus sign.)

2. Changing Mixed Numbers to Improper Fractions: Mixed numbers can be written as improper fractions: To write a mixed number as an improper fraction,

- Multiply the denominator of the proper fraction by the whole number and add the numerator of the proper fraction to your result. This gives you the numerator of the improper fraction
- Carry along the denominator of the proper fraction for the denominator of the improper fraction.

Example: Convert $5\frac{3}{4}$ to an improper fraction.

$$5\frac{3}{4}=\frac{4\bullet 5+3}{4}=\frac{23}{4}$$

Example: Convert each of the following mixed numbers to improper fractions.

a. $5\frac{3}{7}=\frac{5\cdot 7+3}{7}=\frac{35+3}{7}=\frac{38}{7}$

b. $7\frac{5}{8}=\frac{7\cdot 8+5}{8}=\frac{56+5}{8}=\frac{61}{8}$

3. Changing improper fractions to mixed numbers: To change an improper fraction to a mixed number, divide the numerator by the denominator. The quotient is the whole number part of the mixed number, the remainder is the numerator in the mixed number's proper fraction, and the divisor is the denominator in that fraction.

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Example: Convert $\frac{21}{5}$ to a mixed number.

$5\overline{)21}^{\,4}$ with a remainder of 1

so $\frac{21}{5} = 4\frac{1}{5}$

Example: Convert each of the following improper fractions to a mixed number.

a. $\frac{34}{7} = 4\frac{6}{7}$

SDWK

$$\begin{array}{r} 4 \text{ R } 6 \\ 7\overline{)34} \\ -28 \\ \hline 6 \end{array}$$

b. $\frac{19}{2} = 9\frac{1}{2}$

SDWK

$$\begin{array}{r} 9 \text{ R } 1 \\ 2\overline{)19} \\ -18 \\ \hline 1 \end{array}$$

c. $\frac{43}{5} = 8\frac{3}{5}$

SDWK

$$\begin{array}{r} 8 \text{ R } 3 \\ 5\overline{)43} \\ -40 \\ \hline 3 \end{array}$$

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3.7 Multiplication and Division with Mixed Numbers

1. Multiplying and Dividing Mixed Numbers: To multiply or divide mixed numbers, convert each mixed number to an equivalent improper fraction, then complete the multiplication and/or division. Be sure to divide out common factors before you multiply.

Example: Simplify.

$$2\frac{1}{3}\bullet 6\frac{3}{4}=\frac{2\bullet 3+1}{3}\bullet\frac{4\bullet 6+3}{4}$$ convert to improper fractions

$$=\frac{7}{3}\bullet\frac{27}{4}$$ note that 3 is a common factor

$$=\frac{7}{3\div 3}\bullet\frac{27\div 3}{4}$$ divide out common factors

OR use prime factorization & C.C.F.

$$=\frac{7}{1}\bullet\frac{9}{4}=\frac{63}{4}$$ multiply

If your answer is an improper fraction, you may leave it improper or change it to a mixed number, whichever you prefer.

Example: Simplify each of the following.

a. $3\frac{1}{2}\bullet 2\frac{1}{6}=\frac{3\cdot 2+1}{2}\cdot\frac{2\cdot 6+1}{6}$

$=\frac{7}{2}\cdot\frac{13}{6}$

$=\frac{7\cdot 13}{2\cdot 2\cdot 3}$

$=\frac{91}{12}$

b. $2\frac{3}{4}\bullet 3\frac{1}{5}=\frac{2\cdot 4+3}{4}\cdot\frac{3\cdot 5+1}{5}$

$=\frac{11}{4}\cdot\frac{16}{5}$

$=\frac{11\cdot\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2}{\cancel{2}\cdot\cancel{2}\cdot 5}$

$=\frac{11\cdot 2\cdot 2}{5}$

$=\frac{44}{5}$

SDUK

6
2 3

16
2 8
2 4
2 2

4
2 2

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c. $\frac{3}{8} \bullet 5\frac{1}{3} = \frac{3}{8} \cdot \left(\frac{5 \cdot 3+1}{3}\right)$

$= \frac{3}{8} \cdot \frac{16}{3}$

$= \frac{\cancel{3} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 1}$

$= \frac{2}{1}$

$= 2$

d. $\frac{7}{8} \bullet 6 \bullet 5\frac{1}{3} = \frac{7}{8} \cdot \frac{6}{1} \cdot \left(\frac{5 \cdot 3+1}{3}\right)$

$= \frac{7 \cdot 6 \cdot 16}{8 \cdot 1 \cdot 3}$

$= \frac{7 \cdot 2 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 1 \cdot \cancel{3}}$

$= \frac{7 \cdot 2 \cdot 2}{1}$

$= \frac{28}{1}$

$= 28$

e. $1\frac{3}{5} \bullet 2\frac{4}{5} = \left(\frac{1 \cdot 5+3}{5}\right) \cdot \left(\frac{2 \cdot 5+4}{5}\right)$

$= \frac{8}{5} \cdot \frac{14}{5}$

$= \frac{112}{25}$

f. $2\frac{2}{9} \bullet \left(2\frac{1}{4} \div 3\right) = \left(\frac{2 \cdot 9+2}{9}\right) \cdot \left[\left(\frac{2 \cdot 4+1}{4}\right) \div \frac{3}{1}\right]$

$= \frac{20}{9} \cdot \left[\frac{9}{4} \div \frac{3}{1}\right]$

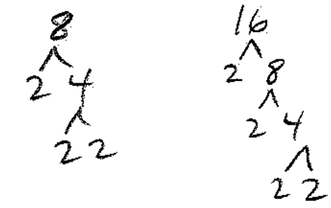
$= \frac{20}{9} \cdot \left[\frac{9}{4} \cdot \frac{1}{3}\right]$

$= \frac{\cancel{2} \cdot \cancel{2} \cdot 5 \cdot \cancel{3} \cdot \cancel{3} \cdot 1}{\cancel{3} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{2} \cdot 3}$

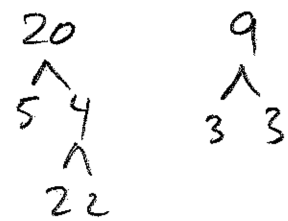
$= \frac{5 \cdot 1}{3}$

$= \frac{5}{3}$

SPWK



$$\begin{array}{r} ^{3}14 \\ \times\ 8 \\ \hline 112 \end{array}$$



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3.8 Addition and Subtraction with Mixed Numbers

1. Adding mixed numbers: Mixed numbers may be added in two ways.

- You may convert all mixed numbers to improper fractions, add the improper fractions, and then convert back to a mixed number.

Example: Simplify.

$$\begin{aligned} 5\frac{3}{4}+9\frac{5}{6} &= \frac{23}{4}+\frac{59}{6} \\ &= \frac{69}{12}+\frac{118}{12} \\ &= \frac{187}{12} = 15\frac{7}{12} \end{aligned}$$

- You may add the whole numbers to get the whole number portion of your answer and then add the proper fractions to get the fraction part of your answer. Sometimes the fraction part turns out to be an improper fraction. When this happens, you must convert the improper fraction to a mixed number and then add the result to the whole number part of your sum.

Example : Simplify.

$$\begin{aligned} 5\frac{3}{4}+9\frac{5}{6} &= 5\frac{9}{12}+9\frac{10}{12} \\ &= 14\frac{19}{12} \\ &= 14+1\frac{7}{12} \\ &= 15\frac{7}{12} \end{aligned}$$

Example: Simplify each of the following. Use either of the methods given above. Be sure to fully reduce all fractions that appear in answers.

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a. $2\frac{2}{9}+3\frac{5}{12}=\frac{2\cdot 9+2}{9}+\frac{3\cdot 12+5}{12}$

$=\frac{20}{9}+\frac{41}{12}$

$=\frac{20}{9}\cdot\frac{4}{4}+\frac{41}{12}\cdot\frac{3}{3}$

$=\frac{80}{36}+\frac{123}{36}$

$=\frac{80+123}{36}$

$=\frac{203}{36}$

OR

$=5\frac{23}{36}$

SDWK

9 = 3·3, 12 = 2·6, 6 = 2·3

$9=3\cdot 3$
$12=2\cdot 2\cdot 3$
$LCD=2\cdot 2\cdot 3\cdot 3$
$LCD=4\cdot 9$
$LCD=36$

$41\times 3=123$

$203 \div 36 = 5\ R23$; $203-180=23$

b. $5\frac{3}{10}+8\frac{11}{15}=5+\frac{3}{10}+8+\frac{11}{15}$

$=13+\frac{3}{10}+\frac{11}{15}$

$=13+\frac{3}{2\cdot 5}\cdot\frac{3}{3}+\frac{11}{3\cdot 5}\cdot\frac{2}{2}$

$=13+\frac{9}{30}+\frac{22}{30}$

$=13+\frac{9+22}{30}$

$=13+\frac{31}{30}$

$=13+1+\frac{1}{30}$

$=14+\frac{1}{30}$

$=14\frac{1}{30}$

SDWK

10 = 2·5, 15 = 3·5

$10=2\cdot 5$
$15=3\cdot 5$
$LCD=2\cdot 3\cdot 5$
$LCD=30$

$31 \div 30 = 1\ R1$; $31-30=1$

2. Subtracting mixed numbers: Mixed numbers may be subtracted in several ways.

- You may change all mixed numbers to improper fractions, then subtract. This method always works.
  Example: Simplify.
  $$4\frac{3}{4}-1\frac{5}{6}=\frac{19}{4}-\frac{11}{6}$$
  $$=\frac{57}{12}-\frac{22}{12}$$
  $$=\frac{35}{12}=2\frac{11}{12}$$

- You may subtract the whole numbers, and subtract the proper fractions. This method is problematic if the second fraction is larger than the first. When this happens, you must use a borrowing technique before you can subtract.

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Example: Simplify.

$$4\frac{3}{4}-1\frac{5}{6}=4\frac{9}{12}-1\frac{10}{12}$$
$$=3\frac{21}{12}-1\frac{10}{12} = 3+\frac{21}{12}-\left(1+\frac{10}{12}\right)$$
$$= 3+\frac{21}{12}-1-\frac{10}{12}$$
$$=2\frac{11}{12}$$

Missing Steps

Example: Simplify each of the following:

a. $7\frac{9}{10}-6\frac{3}{5}$

$$=7\frac{9}{10}-6\frac{6}{10}$$
$$=7+\frac{9}{10}-\left[6+\frac{6}{10}\right]$$
$$=7+\frac{9}{10}-6-\frac{6}{10}$$
$$=(7-6)+\left(\frac{9-6}{10}\right)$$
$$=1+\frac{3}{10}$$
$$=1\frac{3}{10}$$

b. $7\frac{3}{4}-3\frac{5}{12}$

$$=7\frac{9}{12}-3\frac{5}{12}$$
$$=7+\frac{9}{12}-\left(3+\frac{5}{12}\right)$$
$$=7+\frac{9}{12}-3-\frac{5}{12}$$
$$=(7-3)+\left(\frac{9}{12}-\frac{5}{12}\right)$$
$$=4+\frac{4}{12}$$
$$=4+\frac{\cancel{2}\cdot 2\cdot 1}{\cancel{2}\cdot 2\cdot 3}$$
$$=4+\frac{1}{3}$$
$$=4\frac{1}{3}$$

c. $5\frac{1}{3}-3\frac{4}{5}$

$$=\frac{5\cdot 3+1}{3}-\frac{3\cdot 5+4}{5}$$
$$=\frac{16}{3}-\frac{19}{5}$$
$$=\frac{16}{3}\cdot\frac{5}{5}-\frac{19}{5}\cdot\frac{3}{3}$$
$$=\frac{80}{15}-\frac{57}{15}$$
$$=\frac{80-57}{15}$$
$$=\frac{23}{15}$$
$$=1\frac{8}{15}$$

d. $12\frac{3}{4}-5\frac{7}{8}$

$$=\frac{12\cdot 4+3}{4}-\frac{5\cdot 8+7}{8}$$
$$=\frac{51}{4}-\frac{47}{8}$$
$$=\frac{51}{4}\cdot\frac{2}{2}-\frac{47}{8}$$
$$=\frac{102}{8}-\frac{47}{8}$$
$$=\frac{102-47}{8}$$
$$=\frac{55}{8}$$
$$=6\frac{7}{8}$$

SDWK

10 → 2, 5   5 → 1, 5

$10=2\cdot 5$
$5=1\cdot 5$
$LCD=1\cdot 2\cdot 5$
$LCD=10$

$\frac{3}{5}=\frac{3}{5}\cdot\frac{2}{2}=\frac{6}{10}$

4 → 2, 2   12 → 3, 4 → 2, 2

$4=2\cdot 2$
$12=2\cdot 2\cdot 3$
$LCD=2\cdot 2\cdot 3$
$LCD=12$

$\frac{3}{4}=\frac{3}{4}\cdot\frac{3}{3}=\frac{9}{12}$

$LCD=3\cdot 5$
$LCD=15$

$16 \times 5 = 80$ $19 \times 3 = 57$

$23 \div 15 = 1$ R8 ($23-15=8$)

4 → 2, 2   8 → 2, 4 → 2, 2

$4=2\cdot 2$
$8=2\cdot 2\cdot 2$
$LCD=2\cdot 2\cdot 2$
$LCD=8$

$51 \times 2 = 102$ $102-47=55$

$55 \div 8 = 6$ R7 ($55-48$)

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3.9 Combinations of Operations with Fractions

Use the order of operations agreement and the rules of operations with fractions to simplify the following.

a. $4+\left(1\frac{1}{2}\right)\left(2\frac{3}{4}\right)$

$= 4 + \left(\frac{1\cdot 2+1}{2}\right)\cdot\left(\frac{2\cdot 4+3}{4}\right)$

$= 4 + \frac{3}{2}\cdot\frac{11}{4}$

$= 4 + \frac{33}{8}$

$= 4 + 4\frac{1}{8}$

$= 8\frac{1}{8}$ or $\frac{65}{8}$

SDWK

$8\overline{)33}$ = 4 R1

-32

1

$\frac{8\cdot 8+1}{8} = \frac{65}{8}$

b. $50\left(\frac{1}{5}\right)^2 - 12\left(\frac{1}{2}\right)^2$

$= \frac{50}{1}\cdot\frac{1}{25} - \frac{12}{1}\cdot\frac{1}{4}$

$= \frac{2\cdot \cancel{5}\cdot \cancel{5}\cdot 1}{1\cdot \cancel{5}\cdot \cancel{5}} - \frac{\cancel{2}\cdot\cancel{2}\cdot 3\cdot 1}{1\cdot\cancel{2}\cdot\cancel{2}}$

$= \frac{2\cdot 1}{1} - \frac{3\cdot 1}{1}$

$= 2 - 3$

$= -1$

SDWK

$\left(\frac{1}{5}\right)^2 = \frac{1}{5}\cdot\frac{1}{5} = \frac{1}{25}$

$\left(\frac{1}{2}\right)^2 = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

50 → 5, 10 → 2, 5

12 → 2, 6 → 2, 3

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c. $7-\left(1\frac{3}{5}\right)\left(2\frac{1}{2}\right)$

$= 7-\left(\frac{1\cdot5+3}{5}\right)\left(\frac{2\cdot2+1}{2}\right)$

$= 7-\left[\frac{8}{5}\cdot\frac{5}{2}\right]$

$= 7-\frac{2\cdot2\cdot\cancel{2}\cdot\cancel{5}}{\cancel{5}\cdot\cancel{2}\cdot1}$

$= 7-\frac{2\cdot2}{1}$

$= 7-4$

$= 3$

d. $6\left(\frac{1}{2}\right)+\frac{2}{3}(12)$

$= \frac{6}{1}\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{12}{1}$

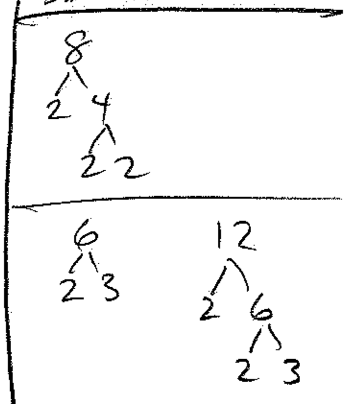
$= \frac{\cancel{2}\cdot3\cdot1}{1\cdot\cancel{2}}+\frac{2\cdot2\cdot2\cdot\cancel{3}}{\cancel{3}\cdot1}$

$= \frac{3\cdot1}{1}+\frac{2\cdot2\cdot2}{1}$

$= 3+8$

$= 11$

SD wK



e. $\frac{2}{5}\left(2\frac{1}{2}\right)+\frac{5}{8}\left(3\frac{1}{5}\right)$

$= \frac{2}{5}\cdot\left(\frac{2\cdot2+1}{2}\right)+\frac{5}{8}\cdot\left(\frac{3\cdot5+1}{5}\right)$

$= \frac{\cancel{2}}{\cancel{5}}\cdot\frac{\cancel{5}}{\cancel{2}}+\frac{5}{8}\cdot\frac{16}{5}$

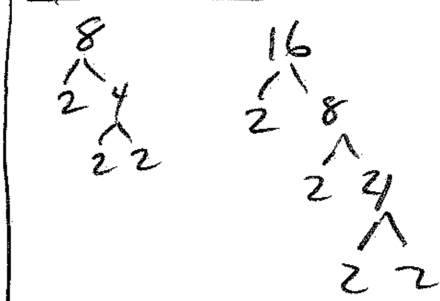
$= 1+\frac{\cancel{5}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot2}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{5}\cdot1}$

$= 1+\frac{2}{1}$

$= 1+2$

$=3$

SD wK



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